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Human capital accumulation, fertility and economic development

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Abstract We examine the development paths of an economy by incorporating the trade-off between the quality and quantity of children and the substitutability between the educational effect within the family and the education paid for by the parent. There is a threshold wage rate, above which individuals begin to invest in the human capital of their children, while reducing the number of children. At this point, the economy shifts from an exogenous growth phase to an endogenous growth phase. It is also shown that the aggregate saving rate is positively correlated with the youth dependency ratio in the development process.

Keywords Human capital · Fertility · Development trap · Economic development · School education

JEL Classification I21 · J13 · O11

1 Introduction

It has often been said that having fewer children enables parents to save more because of declines in rearing costs (e.g., Horioka 1997; Kögel 2005). However, a positive correlation between the saving rate and youth dependency may be often observed in developed countries. Figure 1 shows the household saving rates and the total fertility rates in OECD countries. Declines in the total fertility rate do not necessarily seem to push up the household saving rates about 10–15 years later when newborns reach school age. Our purpose in this study is to suggest parental human capital accumulation

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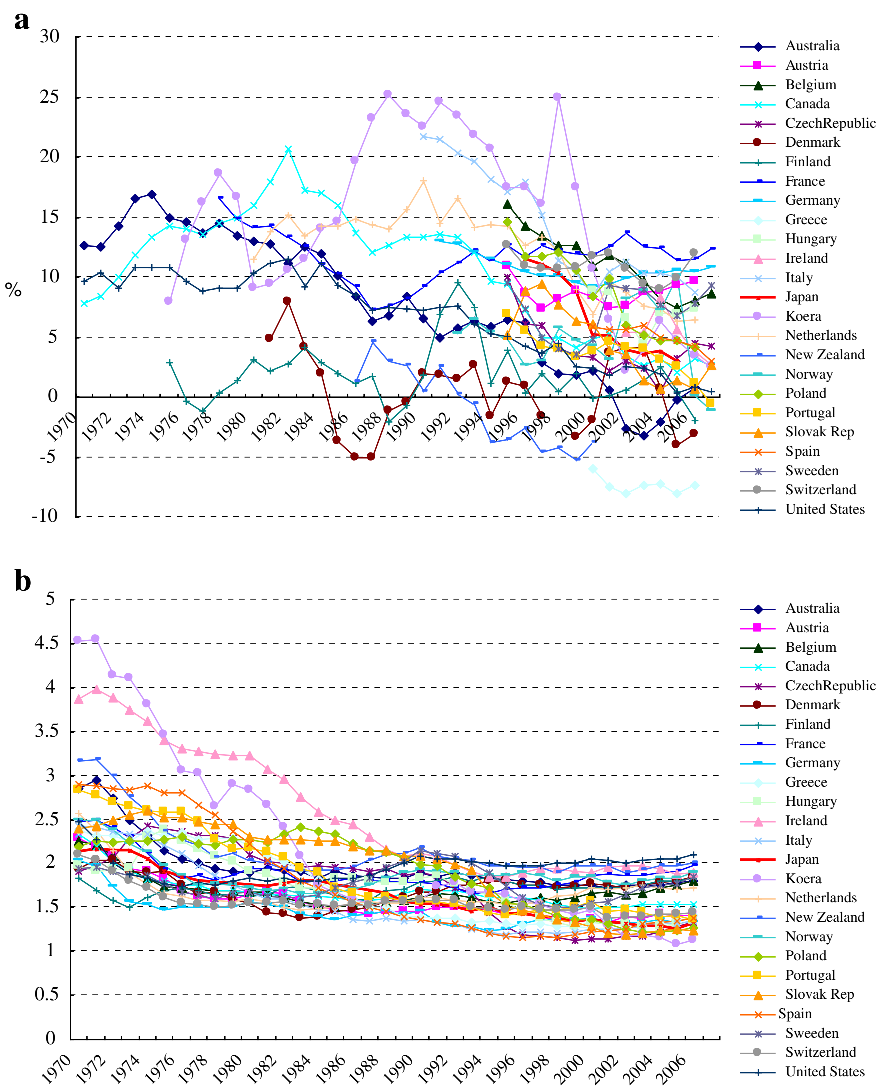


Fig. 1 **a** Household saving rates in OECD countries. **b** Total fertility rates in OECD countries. Source: OECD Factbook 2009

as an explanation for the possible positive correlation between them. In doing so, we derive a development process with economic phase changes owing to changes in the rates of human capital accumulation and population growth.

Now it is widely recognized that human capital accumulation is one of the important engines of economic growth, and a vast literature incorporating the engine has been published since the influential work of Lucas (1988). The extension into an overlapping generations setting was done in studies, for example, by Azariadis and Drazen (1990), Stokey (1991), and Glomm and Ravikumar (1992). On the other hand, the trade-off between the quality and quantity of children has been incorporated into economic models since Becker and Lewis (1973) and Willis (1973) suggested its theoretical possibility.[1] In the recent literature on the growth theory, the effects of fertility dynamics and changes in human capital accumulation and/or research and development (R&D) on economic development have been one of the central issues: Galor and Weil (1999, 2000) and De la Croix and Doepke (2003) among others delivered dynamics in which income growth and fertility can be correlated positively or negatively depending on the economic environment, while many studies, including those of Galor and Moav (2004, 2006), have related the development process to income inequalities. Physical capital accumulated by those with high-saving rates demands education and more stock of human capital per worker because of the capital-skill complementarity.

In the present study, abstracting from income inequality and assuming instead that a new generation starts out with the human capital of an earlier generation, i.e., with learning by watching from the parent, and then parents may invest in augmenting the human capital endowment of their children as school education, we first investigate a development process with changes in human capital accumulation and fertility dynamics. The model used is a two-period-lived overlapping generations model, incorporating both the trade-off between the quality and quantity of children and the substitutability between the intergenerational transmission of human capital within the family and education paid for by the parent. By the term substitutability we mean that either the educational expenditure or the parent's human capital may not be essential inputs in human capital production.[2] The substitutability between education within the family and at school has not been considered in the literature except for De la Croix and Doepke (2003) and Tabata (2003). However, De la Croix and Doepke (2003) concentrated on ergodic growth in the model with heterogeneous agents, while Tabata (2003) introduced the exogenously given innate ability of children which is perfectly substitutable with education paid by the parent. We first construct a simplified endogenous growth version of De la Croix and Doepke (2003) model with homogenous agents, in which parents are altruistic in the sense that they care about not only the quantity (i.e., the number) but also the quality (i.e., education) of their children.[3]

On the other hand, the effects of changes in the population age structure on economic growth have been pointed out empirically. For example, Horioka (1997) illustrated that the youth dependency ratio and the elderly dependency ratio affect the saving rate of

[1] For empirical evidences of the trade-off, see, for example, Hanushek (1992).

[2] This notion may be important especially at earlier stages of development. Morishima (2000) showed that the proportions of business elite from the "Bushi" class, the highest class of the status system in the Edo era (1603–1867), and from those educated in universities were 26 and 13%, respectively, in the 1868–1882 period (in the Meiji era), against 36 and 62%, respectively, in the 1921–1926 period (in the Taisho and Showa era). While the social positions of the Bushi class were defined by their origins and ordered according to their family lines (although their education levels were relatively high), university graduates were from various social classes and they obtained their economic positions by their abilities and efforts. This provides an anecdotal example for the substitutability in human capital production.

[3] De la Croix and Doepke's (2003) main concern is the inequality in human capital among individuals, and they did not examine the dynamics. For other types of formulations for human capital accumulation in overlapping generations settings, see, for example, De la Croix and Michel (2002).

the economy, while Kögel (2005) demonstrated a negative relationship between youth dependency and total factor productivity (TFP) growth and suggested as theoretical background the effect of changes in the youth dependency ratio on aggregate savings. However, neither author considered human capital accumulation explicitly. Our second, but equally important, goal is to derive a theoretical prediction for the relationship between aggregate savings and youth dependency in the presence of parental human capital investments.

The results obtained are as follows. First, in the development process of an economy whose initial level of physical capital stock is sufficiently low, once the wage rate exceeds a threshold value, the economy moves from a phase of exogenous growth to a phase of endogenous growth with an engine of human capital accumulation. The replacement of the growth engine from physical capital accumulation to human capital accumulation is shown to occur without income inequality and resultant leading industrial demand for human capital. However, depending on the values of structure parameters, the economy may fall into a development trap in which parents do not invest in the human capital of their children. We then show that the aggregate saving ratio is positively correlated with the youth dependency ratio in the endogenous growth phase since parents spend more on the education of their children than increasing their own life-cycle savings in contrast to Horioka (1997) and Kögel (2005).

The rest of the paper is organized as follows. The next section introduces a model, and Sect. 3 presents the dynamics of the system. We examine the development process of the economy in Sect. 4, and Sect. 5 relates our results to the preceding literature on the relationship between the age structure of the population and the saving rate of the economy. Section 6 concludes the paper.

2 The model

Each individual lives for three periods; childhood, adulthood and old age. He receives education in the first period, works and rears children in the second, and retires in the third. The labor productivity of an individual is represented by the stock of human capital he has at the beginning of the working period. We assume that the parental education consists of two parts: one is financed by parental expenditure and the other is direct educational effects from the child's parent, i.e., intergenerational transmission of human capital within the family.[4] The direct effect is assumed to depend on the stock of human capital of the individual's parent, while the educational expenditure is chosen by the parent. Firms produce output using physical capital and effective labor under constant-returns-to-scale production technology. Physical capital depreciates completely after a one-period use in production.

2.1 Individuals

The length of each period of an individual's life is fixed and normalized to one. In the second period of his life, an individual divides his time between working and rearing

[4] In this study we assume unisex individuals as usual in the literature.

children. We assume that rearing time per child is constant. He also spends on the education of his children in the period. Consumption during retirement in the third period is financed by the returns to savings accumulated in the second period. For simplicity we omit consumption in the second period in the present study.

The human capital of an individual working in period $t+1$ is assumed to be produced as follows:

$$h_{t+1} = \varepsilon(e_t + \theta h_t)^{\delta} \bar{h}_t^{1-\delta} \quad 0 < \delta \leq 1;\ 0 < \varepsilon, \theta \tag{1}$$

where h_t is the stock of human capital of an individual of the working generation in period t (which we call generation t), e_t is per child educational expenditure by his parent, and $\bar{h}_t$ is the average stock of human capital of generation t.[5] The term $e_t + \theta h_t$ denotes education provided for the individual within the family where θ may reflect the ability of children to absorb the human capital of their parents: One component is education paid for by the parent, and the other is the educational effect obtained by watching from the parent, reflecting family and/or parental background. Here, the relative efficiency of educational expenditure in human capital production is normalized to one. In our formulation, since they are perfect substitutes, either the educational expenditure or the parent's human capital is not really an essential input in human capital production, i.e., children need not be sent to school if they can learn enough at home or the children can be educated at school if their parents are not educated.[6] On the other hand, $\bar{h}_t^{1-\delta}$ represents the spillovers from society, reflecting the fact that learning is more productive if an individual interacts with more knowledgeable people.[7]

Parental altruism reflects the fact that parents affect the welfare of their children primarily by influencing their potential earnings (see Becker and Tomes 1986; Glomm and Kaganovich 2008). The lifetime utility of an individual of generation t is therefore assumed to be represented by a log linear function:

$$U_t = \rho \ln c_{t+1} + \gamma \ln n_t + \beta \ln h_{t+1} \quad \rho, \gamma, \beta > 0$$

where c_{t+1} is his consumption during retirement and n_t is the number of his children. Denoting per child rearing time, the wage rate for effective labor and life-cycle savings as $z\ (>0)$, w_t and s_t, respectively, the budget constraint of the individual during the working period is given as

$$w_t h_t (1 - z n_t) = s_t + e_t n_t \tag{2}$$

[5] As in Rangazas (2000), the human capital production function of homogeneous of degree one is compatible with endogenous growth.

[6] The intergenerational educational effect may not be substituted with parental education investments for each other at lower wage rates because of the non-negative constraint $e_t \geq 0$, although a greater θ per se tends to reduce parental expenditures on education in our model.

[7] The externalities in human capital accumulation can be only local.

and, letting the interest rate in period $t+1$ be r_{t+1}, the third-period budget constraint is

$$c_{t+1} = r_{t+1}s_t. \tag{3}$$

The problem for the individual of generation t is to choose consumption c_{t+1}, the number of children n_t, and the educational expenditure e_t so as to maximize his lifetime utility U_t. The first-order conditions for maximization give the following:[8]

$$s_t = \frac{\rho}{\rho+\gamma} w_t h_t \tag{4}$$

$$n_t = \begin{cases} \frac{\gamma}{\rho+\gamma}\frac{1}{z} & w_t \leq \frac{\gamma\theta}{\beta\delta z} \\ \frac{\gamma-\beta\delta}{\rho+\gamma}\frac{w_t}{w_t z-\theta} & w_t > \frac{\gamma\theta}{\beta\delta z} \end{cases} \tag{5}$$

$$e_t = \begin{cases} 0 & w_t \leq \frac{\gamma\theta}{\beta\delta z} \\ \frac{\beta\delta w_t z-\gamma\theta}{\gamma-\beta\delta} h_t & w_t > \frac{\gamma\theta}{\beta\delta z} \end{cases} \tag{6}$$

where $dn_t/dw_t < 0$ and $de_t/dw_t > 0$ for $w_t > \gamma\theta/\beta\delta z$. There is a critical wage rate at which an individual starts investment in the quality of children, $\gamma\theta/\beta\delta z$. The optimizing behavior can be interpreted as follows: The marginal benefit of additional education per child to the marginal cost at $e_t = 0$ is $(dU_t/dh_{t+1})(dh_{t+1}/de_t) = \beta\delta/\theta h_t$, while the marginal cost is $d(e_t n_t)/de_t = n_t$. Therefore, the ratio of the marginal benefit to the marginal cost is given as $(\beta\delta/\theta h_t)/n_t$ at $e_t = 0$. On the other hand, the ratio of the marginal benefit of an additional child to the marginal (opportunity) cost is given as $(dU_t/dn_t)/(w_t h_t z) = (\gamma/n_t)/(w_t h_t z)$. If the former is smaller than the latter, i.e., if $w_t \leq \gamma\theta/\beta\delta z$, the individual prefers to have children rather than spend on their education without changing the number of children they have. In contrast, if the wage rate is above the threshold value $\gamma\theta/\beta\delta z$, the individual is likely to spend on education of children than have more children. For the wage rate greater than the critical value, other things being equal, the higher the wage rate, the more parents spend on their children's education. The tradeoff between quality and quantity of children is essentially the same as that in De la Croix and Doepke (2003), in which the critical value is given for human capital of an individual relative to the average stock in the economy. We assume here that $(\gamma-\beta\delta)/[(\rho+\gamma)z] \geq 1$. If this condition fails to hold, the population may approach zero as the wage rate rises to infinite.

Now, since $h_t = \bar{h}_t$ from the assumption of identical individuals within a generation, we have from (1) and (6):

$$h_{t+1} = \begin{cases} \varepsilon\theta^{\delta} h_t & w_t \leq \frac{\gamma\theta}{\beta\delta z} \\ \varepsilon(\beta\delta)^{\delta}\left(\frac{w_t z-\theta}{\gamma-\beta\delta}\right)^{\delta} h_t & w_t > \frac{\gamma\theta}{\beta\delta z} \end{cases} \tag{7}$$

[8] From the second-order condition, we have $\gamma - \beta\delta > 0$. It should be noted that this condition is satisfied even if $\gamma = \beta$, as in De la Croix and Doepke (2003).

Thus, when the wage rate is sufficiently low, i.e., when $w_t \le \gamma\theta/\beta\delta z$, individuals do not spend on the education of their children, and the rate of change in per worker stock of human capital is constant from generation to generation, i.e., $h_{t+1}/h_t = \varepsilon\theta^{\delta}$, while the rate of change in human capital depends on the wage rate when the wage rate is high, i.e., when $w_t > \gamma\theta/\beta\delta z$ is satisfied. Especially, if $\varepsilon\theta^{\delta} = 1$, the *level* of per worker stock of human capital remains constant from generation to generation.[9]

2.2 Production

Assuming that there are many competitive producers with the constant-returns-to-scale production technology, the aggregate technology of the economy can be represented by the following production function:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1 \tag{8}$$

where Y_t, K_t and L_t are aggregate output, physical capital and effective labor employed in period t, respectively. The profit maximization conditions are given as

$$\alpha(K_t/L_t)^{\alpha-1} = r_t \tag{9a}$$

$$(1-\alpha)(K_t/L_t)^{\alpha} = w_t \tag{9b}$$

2.3 Marker equilibrium

The equilibrium condition in the capital market is given as

$$s_t N_t = K_{t+1} \tag{10}$$

where N_t is the population of generation t. The equilibrium condition in the labor market may be written as

$$L_t = (1 - z n_t) h_t N_t \tag{11}$$

From the budget constraints of each generation (2) and (3) (with a one-period lag), the profit maximization condition and the zero profit condition (9), and the equilibrium conditions in the capital and labor markets (10) and (11), we obtain the resource constraint in period t:

$$Y_t = c_t N_{t-1} + e_t n_t N_t + K_{t+1} \tag{12}$$

Output is allocated among individual consumption, educational expenditure for children and capital accumulation for the next period.

9 Bovenberg and van Ewijk (1997) and Yakita (2003) assumed that the "socially" endowed portion of human capital (i.e., $\varepsilon\theta^{\delta}$ in our notation) is equal to or smaller than unity, while Meijdam (1998) considered a case in which it is greater than one. For intergenerational transmission of human capital, see also Stokey (1991).

3 Dynamics: two phases

The dynamic system of this model is given by three equations (7), (10) and $N_{t+1} = n_t N_t$, with three state variables, h_t, K_t and N_t. As can be seen from (5) and (7), the dynamics of the system can be separated into two phases: (I) $w_t \leq \gamma\theta/\beta\delta z$ and (II) $w_t > \gamma\theta/\beta\delta z$. We examine them in turn.

Phase (I): $w_t \leq \gamma\theta/\beta\delta z$

Taking into account the utility maximization condition of individuals (4), the profit maximization condition (9b), and the equilibrium condition in the labor market (11), the equilibrium condition in the capital market (10) can be rewritten as

$$k_{t+1} = \frac{\rho(1-\alpha)}{\rho+\gamma}\left(\frac{k_t}{h_t}\right)^{\alpha-1}(1-zn_t)^{-\alpha}n_t^{-1}k_t \tag{13}$$

where $k_t = K_t/N_t$ is per worker stock of physical capital in period t. Since parents do not spend on the education of their children in this phase, we have $h_{t+1} = \varepsilon\theta^\delta h_t$. Since the number of children per parent is constant, we define

$$n = \frac{\gamma}{\rho+\gamma}\frac{1}{z} \tag{14}$$

Thus, from (13) and (14), we obtain the difference equation of the physical capital/human capital ratio, $k_t/h_t = \nu_t$, as follows:

$$\nu_{t+1} = \frac{(1-\alpha)\left(\frac{\rho}{\rho+\gamma}\right)^{1-\alpha}\frac{(\rho+\gamma)z}{\rho}}{\varepsilon\theta^\delta}\nu_t^\alpha \tag{15}$$

We can show that there is a unique stationary solution to Eq. (15), and that the condition $0 < d\nu_{t+1}/d\nu_t < 1$ holds in the stationary solution. The stationary solution ν^* can be given as

$$\nu^* = \left[\frac{(1-\alpha)\left(\frac{\rho}{\rho+\gamma}\right)^{1-\alpha}\frac{(\rho+\gamma)z}{\rho}}{\varepsilon\theta^\delta}\right]^{1/(1-\alpha)} \tag{16}$$

Phase (II): $w_t > \gamma\theta/\beta\delta z$

When the wage rate becomes sufficiently high, individuals begin to spend on the education of children, while reducing their number. Therefore, the rate of change in per worker stock of human capital varies from generation to generation. As in the previous case, from (7) and (13), we have the following difference equation of the physical capital/human capital ratio:

$$v_{t+1}=\frac{\frac{\rho(1-\alpha)}{\rho+\gamma}(1-zn_t)^{-\alpha}n_t^{-1}}{\varepsilon\left(\frac{\beta\delta}{\gamma-\beta\delta}\right)^{\delta}(w_tz-\theta)^{\delta}}v_t^{\alpha} \tag{17}$$

where, from (9b) and (11), we have

$$w_t=(1-\alpha)(1-zn_t)^{-\alpha}v_t^{\alpha} \tag{18}$$

$$n_t=\frac{\gamma-\beta\delta}{\rho+\gamma}\frac{w_t}{w_tz-\theta} \tag{5$'$}$$

If the labor market is in equilibrium, the wage rate w_t and the fertility rate n_t must satisfy (5$'$) and (18), simultaneously, for a given value of v_t. That is, from

$$w_t=(1-\alpha)\left(1-z\frac{\gamma-\beta\delta}{\rho+\gamma}\frac{w_t}{w_tz-\theta}\right)^{-\alpha}v_t^{\alpha}$$

we can obtain the wage rate as a function of the physical capital/human capital ratio v_t, i.e., $w_t=w(v_t)$, and, correspondingly, the fertility rate as $n_t=\tilde{n}(w(v_t))=n(v_t)$. Making use of (5$'$) and (18), we obtain

$$\frac{dw_t}{dv_t}=D^{-1}\alpha(1-\alpha)(1-zn_t)^{-\alpha}v_t^{\alpha-1}>0 \tag{19}$$

$$\frac{dn_t}{dv_t}=-D^{-1}\frac{\phi(\gamma-\beta\delta)}{\rho+\gamma}\frac{\theta}{(w_tz-\theta)^2}\alpha(1-\alpha)(1-zn_t)^{-\alpha}v_t^{\alpha-1}<0 \tag{20}$$

where $D=1+\alpha(1-\alpha)z(1-zn_t)^{-\alpha-1}v_t^{\alpha}\frac{\gamma-\beta\delta}{\rho+\gamma}\frac{\theta}{(w_tz-\theta)^2}>1$. The wage rate and the fertility rate are both monotonic in the physical capital/human capital ratio, and an increase in the physical capital/human capital ratio raises the wage rate and reduces the fertility rate. Thus, we may also describe the dynamics of the system in terms of one variable, v_t, in this phase.

Now, we examine the dynamic path of the physical capital/human capital ratio from (17). Differentiating both sides of (17) with respect to v_t, we obtain the following equation:

$$\begin{aligned}\frac{dv_{t+1}}{dv_t}&=\frac{\frac{\rho(1-\alpha)}{\rho+\gamma}}{\varepsilon\left(\frac{\beta\delta}{\gamma-\beta\delta}\right)^{\delta}}\frac{(1-zn_t)^{-\alpha-1}n_t^{-2}v_t^{\alpha-1}}{(w_tz-\theta)^{\delta+1}}\\&\quad\times\left\{-\delta z(1-zn_t)n_tv_t\frac{dw_t}{dv_t}+(w_tz-\theta)v_t[zn_t(1+\alpha)-1]\frac{dn_t}{dv_t}\right.\\&\quad\left.+\alpha(w_tz-\theta)(1-zn_t)n_t\right\}\\&=\frac{\frac{\rho(1-\alpha)}{\rho+\gamma}}{\varepsilon\left(\frac{\beta\delta}{\gamma-\beta\delta}\right)^{\delta}}\frac{(1-zn_t)^{-\alpha}n_t^{-1}v_t^{\alpha-1}}{(w_tz-\theta)^{\delta+1}}D^{-1}(1-\delta)\alpha w_tz>0\end{aligned} \tag{21}$$

That is, the physical capital/human capital ratio changes monotonically. The stationary-state physical capital/human capital ratio can be obtained from (17) as

$$v_\infty = \left[\frac{\frac{\rho(1-\alpha)}{\rho+\gamma}(1 - zn_\infty)^{-\alpha} n_\infty^{-1}}{\varepsilon \left(\frac{\beta\delta}{\gamma-\beta\delta}\right)^\delta (w_\infty z - \theta)^\delta} \right]^{\frac{1}{1-\alpha}} \tag{22}$$

where $w_\infty = (1-\alpha)(1-zn_\infty)^{-\alpha} v_\infty^\alpha$ and $n_\infty = \frac{\gamma-\beta\delta}{\rho+\gamma} \frac{w_\infty}{w_\infty z - \theta}$. We have three equations for three variables $(v_\infty, w_\infty, n_\infty)$ in characterizing a stationary solution of the system. To distinguish from the steady state without education investment, we call the stationary state v_∞ the balanced growth equilibrium in the following. Using (22), condition (21) can be rewritten as

$$\frac{dv_{t+1}}{dv_t} = D^{-1} \frac{(1-\delta)\alpha}{1 - (\theta/w_\infty z)} \tag{23}$$

Since $D > 1$, a sufficient condition for the stability is $(1-\delta)\alpha \leq 1-(\theta/w_\infty z)$, which is likely to be satisfied in most plausible cases. For example, setting $(\rho, \alpha, \gamma, \beta, z, \theta, \delta) = (0.366, 0.333, 0.08, 0.08, 0.075, 0.0119, 0.635)$ as those in De la Croix and Doepke (2003) and for convenience $\varepsilon = 24.25$, we have $\theta/w_\infty z = 0.2963$ and $(1-\delta)\alpha = 0.1216$, where we choose the same value for γ and β (as in De la Croix and Doepke 2003) such that $n_\infty[=1.2405 \approx 1.0087^{25}] > 1$ holds, and the balanced growth equilibrium is characterized by triplet $(v_\infty, w_\infty, n_\infty) = (0.0798, 0.2969, 1.2405)$. We assume that the stability condition holds at the stationary equilibrium, although the balanced growth equilibrium can be unstable if the contribution of education in human capital production, δ, is sufficiently small, if the relative contribution of the human capital of parent in education, θ, is sufficiently great, and/or if rearing-time per child, z, is sufficiently small.

4 Endogenous growth and development trap

For an expositional purpose, we assume that $\varepsilon\theta^\delta = 1$ in this section. In this case, per worker stock of human capital remains constant from generation to generation without educational expenditures of individuals (i.e., $h_{t+1}/h_t = 1$). In phase (I) in which the wage rate for effective labor is sufficiently low and individuals do not spend on the education of children, the wage rate rises as physical capital accumulates. Starting from a sufficiently low physical capital/human capital ratio, the economy approaches the steady state v^*, following the difference equation (15). If the steady-state wage rate $w^* = (1-\alpha)[\rho/(\rho+\gamma)]^{-\alpha}(v^*)^\alpha$, which would obtain with the steady-state physical capital/human capital ratio, is lower than the threshold $\gamma\theta/\beta\delta z$, the economy stays in the steady state with the physical capital/human capital ratio v^*. The steady state has the same properties as the steady state in the neoclassical exogenous growth theory, and, therefore, the growth rate of per capita income is zero.[10] The growth rate of effective labor is equal to that of the population, which is given by (14).[11]

In contrast, if the steady-state wage rate w^*, which would obtain in phase (I), is higher than the threshold $\gamma\theta/\beta\delta z$, the economy starting from a sufficiently low physical capital/human capital ratio, will not reach the steady state of phase (I), ν^*. Since individuals start to invest in the human capital of their children after the wage rate becomes greater than the threshold $\gamma\theta/\beta\delta z$, the economy switches to phase (II). That is, if the condition

$$(1-\alpha)\left(\frac{\rho}{\rho+\gamma}\right)^{-\alpha}\left[\frac{(1-\alpha)\left(\frac{\rho}{\rho+\gamma}\right)^{1-\alpha}\frac{(\rho+\gamma)z}{\rho}}{\varepsilon\theta^{\delta}}\right]^{\frac{\alpha}{1-\alpha}} > \frac{\gamma\theta}{\beta\delta z}$$

holds or, equivalently in terms of the physical capital/human capital ratio, if the condition

$$\nu^* = \left[\frac{(1-\alpha)\left(\frac{\rho}{\rho+\gamma}\right)^{1-\alpha}\frac{(\rho+\gamma)z}{\rho}}{\varepsilon\theta^{\delta}}\right]^{\frac{1}{1-\alpha}} > \left[\frac{\gamma\theta}{\beta\delta z(1-\alpha)}\right]^{\frac{1}{\alpha}}\frac{\rho}{\rho+\gamma} \equiv \nu^c \tag{24}$$

holds, the economy shifts from phase (I) to phase (II), where ν^c is the physical capital/human capital ratio which equates the wage rate with $\gamma\theta/\beta\delta z$ when the rate of population growth is n. It should be recalled here that the dynamics of the physical capital/human capital ratio ν_t gives the dynamics of the wage rate w_t (see (9b), (11), (18) and (19)).

The time paths are illustrated in Fig. 2a and b, respectively. When $\nu^c > \nu^*$ as is depicted in Fig. 2a, the economy starting from a sufficiently low physical capital/human capital ratio, say, ν_0, converges monotonically to the steady state S following the dynamics in (15) as time passes. In contrast, Fig. 2b illustrates a case $\nu^c < \nu^*$, in which the economy follows the time path as in phase (I) as long as the physical capital/human capital ratio is smaller than ν^c; but above ν^c the economy goes into phase (II) and evolves in accordance with the dynamics in (17). After a sufficient time passes, the economy approaches the balanced growth equilibrium E. The time path in this case has the same properties as the transition to the steady state in the neoclassical *exogenous* growth as long as $\nu_t \le \nu^c$, and the per capita income growth is exclusively due to physical capital accumulation. However, as the wage rate rises, the physical capital/human capital ratio becomes greater than the critical level, i.e., $\nu_t > \nu^c$. At that point in time, the growth is *endogenously* driven by the engine of human capital accumulation, and, the economy moves towards the balanced growth equilibrium E. At the balanced growth equilibrium, physical capital, human capital

[10] We use the term "exogenous" in the sense that the growth rate of human capital is exogenously given, recalling the assumption of $\varepsilon\theta^{\delta} = 1$. Our analysis is valid with proper modifications even for other cases.

[11] We can not a priori rule out the possibility that the system does not have a balanced growth equilibrium (of type E in Fig. 2) at which individuals invest in human capital.

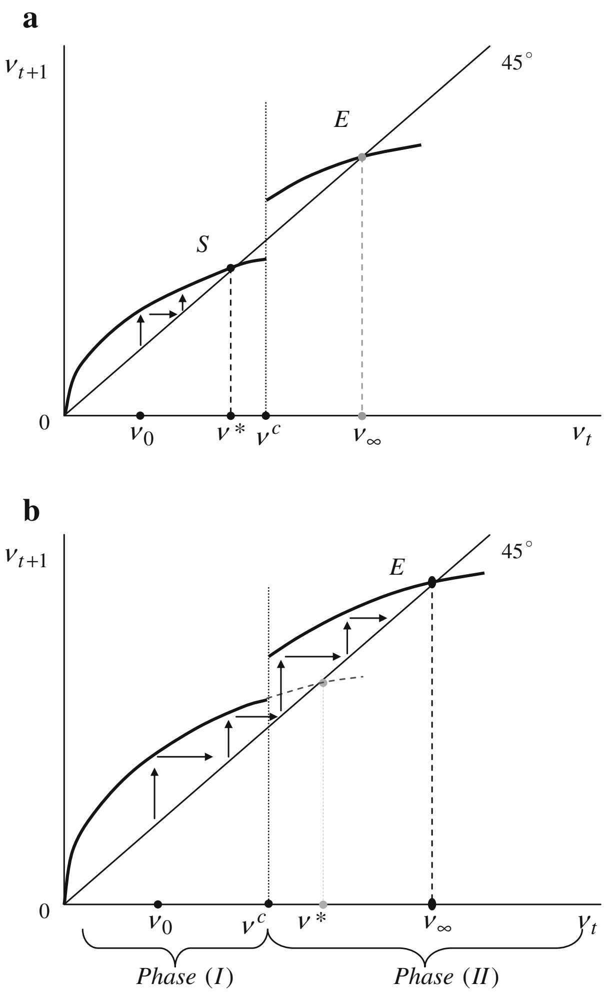


Fig. 2 **a** Case: $\nu^* < \nu_\infty$. **b** Case: $\nu^* > \nu_\infty$

and income, respectively, in per worker terms grow at the same balanced growth rate. The balanced growth rate plus one is given as[12]

$$\frac{k_{t+1}}{k_t} = \frac{h_{t+1}}{h_t} = \varepsilon(\beta\delta)^\delta \left(\frac{w_\infty z - \theta}{\gamma - \beta\delta}\right)^\delta \tag{25}$$

where the population grows at rate $n_\infty - 1$ and aggregate physical capital grows at rate $n_\infty(k_{t+1}/k_t) - 1$. On the transition path converging to the balanced growth

[12] We can readily show that the balanced growth rate (25) is higher than the growth rate without human capital investment.

equilibrium E, the physical capital/human capital ratio is rising, the wage rate for effective labor is increasing (see (19)), and the fertility rate is declining (see (20)).[13]

We obtain the following proposition:

Proposition 1 *In an economy whose initial physical capital/human capital ratio is sufficiently low*:

(i) *If condition* (24) *is satisfied, per capita income grows due to physical capital investment in earlier stages of economic development, but as the wage rate increases, individuals start to invest in human capital and the economy shifts to an endogenously growing path driven by human capital investment. On the transition to the balanced growth path, the fertility rate is declining and the wage rate is increasing.*

(ii) *If condition* (24) *is not satisfied, in contrast, the economy has increases in per worker income owing to physical capital investment in earlier stages, but, in the long term, it only approaches a steady state in which individuals do not invest in human capital and the fertility rate is relatively high. In other words, the economy falls into a development trap.*

The result that an economy may go into balanced growth trough regime changes or fall into a development trap is not necessarily novel. For example, Becker et al. (1990), focusing on human capital accumulation, showed that the economy falls into a development trap with low education and high fertility if the initial stock of human capital is small, while it approaches a balanced growth path with high education and low fertility if the initial stock of human capital is sufficiently great. However, the regime change can be brought about only by an exogenous shock to the human capital stock in their model. On the other hand, recent theoretical literature models economic development process with endogenous regime changes, emphasizing the crucial role of income inequality (e.g., Galor and Moav 2004, 2006; De la Croix and Doepke 2003). Galor and Moav (2004) among others showed that in earlier stages of economic development, inequality channels resources towards individuals with a higher propensity to save, and that the accumulation of physical capital increases the demand for human capital and induces human capital accumulation.[14]

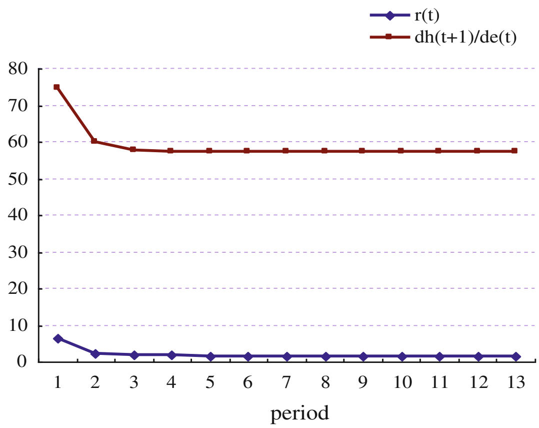
In a model with identical individuals, we showed the possibility of economic development through (endogenous) regime changes regardless of its initial condition only as long as the parameters satisfy certain conditions.[15] The increased wage rate due to physical capital accumulation increases the marginal benefit from educational investment in children relative to the marginal cost (which is reflected in the marginal utility of consumption) on the one hand, and it increases the opportunity costs of child rearing

[13] Assuming the subsistence level of consumption as in Tabata (2003), we can obtain inverted U-shape fertility dynamics. Hazan and Zoabi (2006) also obtained the inverted U shape by incorporating explicitly the impact of children's health on their education.

[14] Although they mentioned that the relationship between income inequality and growth is empirically inconclusive and controversial.

[15] Since our result holds even when $\varepsilon\theta^{\delta} > 1$, this amounts to a change in the rate of human capital accumulation. For the implications of the unified growth theory on macroeconomics, see Galor (2005, 2007).

Fig. 3 Rate of return of educational expenditure and interest rate



time on the other. Thus, in our model, the increased wage income induces individuals to invest in the human capital of their children, and reducing their number. In other words, at the beginning of phase (II), the rate of return of educational expenditure will be high, inducing parents to invest in human capital of their children. The speed of human capital accumulation is faster than that of physical capital. This makes the interest rate, i.e., the rate of return to physical capital, higher than otherwise without the phase change. The increase in effective labor of a highly educated population, whose size is smaller, may need more accumulation of physical capital to be associated with highly educated labor, thus making effective wages higher. Therefore, the rate of return to educational expenditure and the rate of interest are both falling and converge to the balanced growth equilibrium levels, respectively, while the wage rate increases to the equilibrium level.[16] Thus, human capital accumulated by educational expenditure induces physical capital accumulation, which in turn raises the wage rate and also educational expenditures. Figure 3 depicts the time paths of the rate of return of educational expenditure to parents and the interest rate predicted in the numerical example given at the end of Sect. 3 and starting from the initial condition $(k, h) = (0.01, 1)$.[17]

It should be noted that in the transition of phase (II), the abundant supply of human capital induces physical capital accumulation to utilize highly educated labor. This accumulation process of both capitals may be analogous to the directed technical change argued by, for example, Acemoglu (1998) and Caselli and Coleman (2006), although in our model workers are homogeneous and the accumulation of other production factors is accelerated to utilize the abundant factor instead of induced technical changes. This is in contrast to the prediction in Galor and Moav (2004) who stressed the capital-skill complementarity in output production and industrial

16 From the first-order optimal conditions for individuals, we have $dh_{t+1}/de_t = [(\rho n_t h_{t+1})/(\beta c_{t+1})]r_{t+1}$. Although we can not determine the time path of the coefficient of r_{t+1} *a priori*, we can show that it increases in the transition from the phase change in our model with parameters given at the end of Sect. 3. This implies that human capital increases much faster than consumption.

17 In this example, we have $v^c = 0.00733 < v^* = 0.01238$.

demand for human capital induced by physical capital accumulation in earlier stage of the modern growth regime.[18] In other words, while Galor and Moav (2004) proposed a theory characterizing the replacement of physical capital accumulation by human capital accumulation as a prime engine of development from the demand side aspect, we suggest another scenario for the replacement from the supply side perspective.[19]

On the other hand, if the steady-state wage rate does not reach the threshold (which is dependent upon various parameters of the economy), the economy can not reach the transition to the balanced equilibrium growth and falls into a development trap. It should be noted, however, that whether the economy falls into the trap depends not only on the initial value of the physical capital/human capital ratio (i.e., the (reduced) state variable) but also on the parameters of the economy. Condition (24) indicates that the economy is likely to fall into the development trap, other things being equal, if the parameters are such that human capital investment tends to be discouraged, that is, if: (i) per child rearing time z is less, (ii) the utility weight on human capital per child β is smaller, (iii) the saving rate $\rho/(\rho+\gamma)$ is greater, (iv) the scale parameter in human capital production ε is greater, and/or (v) the educational effect of the parent on children θ is greater.

The possibility of falling into the development trap gives some policy implications: First, if parents believe that their children are well educated within the family, owing to a greater θ and/or a small β, and do not send them to school, it may be necessary to give parents incentives to let their children attend school for the economy to take-off, for example, by subsidizing school education at rate τ. With the subsidies the education cost for parents drops to $(1-\tau)e_t n_t$, and the critical wage rate becomes lower, i.e., $(1-\tau)\gamma\theta/\beta\delta z$. On the other hand, the higher the efficiency of educational expenditure, for example $\phi > 1$, the lower the threshold wage rate $\gamma\theta/\beta\delta\phi z(< \gamma\theta/\beta\delta z)$. If the efficiency of school education can be improved, not only is the economy likely to take off, but the human capital investment rate out of the full wage income $(w_t h_t)$ will be higher and so is the balanced growth rate. If physical capital increases sufficiently through ODA from foreign countries, the enhanced physical capital raises the wage rate and may lead the economy to the endogenous growth phase.[20] An intergenerational redistribution policy from retired to working population increases fertility but does not have a direct effect on educational expenditure. These are implications of our results on educational policy in developing economies.

[18] Endogenous fertility in the trade-off between the quality and quantity of children is essential in our study, although Galor and Moav (2004) suggested it as an extension of their model. Lord and Rangazas (2006) suggested that the schooling of older children can be a dominant factor in explaining the fertility decline during the 20th century in the US, while introducing child labor regulations can be important for the fertility decline in England after 1840, as per Doepke (2004).

[19] Endogenous fertility decisions as the trade-off between quality and quantity of children is essential to regime change in our model in contrast to Galor and Moav (2004), who suggest the introduction of endogenous fertility as an extension of their model.

[20] The amount of physical capital given should be adequate. If not, the economy converges back to the steady state, the development trap.

5 Aggregate saving rate and human capital accumulation

Now we examine the relation between the youth dependency ratio, defined as the child population/working population ratio, and the aggregate saving rate of the economy. From (4) and the homogeneity of degree one of the production function, the aggregate saving rate in period t can be written:[21]

$$\frac{s_t N_t}{Y_t} = \frac{\frac{\rho}{\rho+\gamma} w_t h_t}{w_t h_t (1 - z n_t) + r_t k_t} = \frac{\rho/(\rho+\gamma)}{(1 - z n_t)[1 + \alpha/(1-\alpha)]} \tag{26}$$

That is, the aggregate saving rate increases with the fertility rate. A decline in the fertility rate implies not only a fall in the youth dependency ratio since their consumption and educational expenditure are financed by their parental income, but also, if it lasts for a long period, an increase in the elderly dependency ratio, which is defined as the retired population/working population ratio. As can be seen from (26), the aggregate saving rate in period t depends on the fertility rate of generation t (which is equal to the youth dependency ratio in period t in our study), but not on the elderly dependency ratio $(n_{t-1})^{-1}$. That is, the elderly dependency ratio does not affect the aggregate saving rate of the economy.[22]

If the economy is in phase (I), as can be seen from (26), the aggregate saving rate does not change since the fertility rate is constant. In contrast, however, if the economy is in phase (II), the fertility rate and the youth dependency ratio decline monotonically on the transition to a balanced equilibrium growth (E in Fig. 2b).[23] Therefore, we have the following result:

Proposition 2 *The aggregate saving rate falls during the transition to the stationary state, during which the youth dependency ratio is also declining.*

This result of the positive correlation between the saving rate and the youth dependency, however, contrasts with the empirical results of Horioka (1997) and Kögel (2005). Horioka (1997) conjectured that those who have not yet begun working consume but do not earn income and hence that a fall in the youth dependency ratio will have a positive impact on the aggregate saving rate. Kögel (2005) asserted that the increased savings rate due to declines in the fertility rate enables more resources to be devoted to R&D investment and hence will have a positive effect on the TFP growth rate. Horioka (1997) also suggested that the higher the ratio of the retired population to the working population, the lower will be the aggregate saving rate, and Kögel (2005)

[21] Y_t is net disposable income of the household sector, as can be seen from (12), while $s_t N_t$ is net household saving, i.e., the sum of $[w_t h_t (1 - z n_t) - e_t n_t] N_t = s_t N_t$ of the working generation and $(r_t k_t - c_t) N_{t-1} = 0$ of the retired.

[22] This result may depend on the assumption of three periods in a lifetime. That is, individuals accumulate savings only in the second period and they dis-save entirely in the third period.

[23] Including utility from consumption during the working period in a log-linear fashion does not essentially alter our results.

seems to take the time path of population growth as exogenously given when stating that the youth dependency ratio reduces "residual" growth.

In the present study, however, although low fertility leaves more resources at the disposal of parents, the resources will be devoted to educational expenditure on their children rather than savings for their own retirement. Given that individuals derive utility from the quantity and quality of children, our result can be rather natural. In phase (II) of our model, an increase in the wage rate reduces the fertility rate (and thereby the youth dependency ratio). In this case, owing to decreases in the number of children, the working hours of parents increase, and therefore their wage income rises greatly. However, parents increase investments in the human capital of their children rather than increasing their life-cycle savings. The increasing human capital investment on the growth path stems from the fact that human capital is embodied in the individual and that at the individual level human capital accumulation is subject to diminishing returns, while an additional unit of goods brings about per unit consumption in a one-by-one manner. As shown in the previous section, faster accumulation of human capital requires a higher rate of change in educational expenditures. As a consequence, the aggregate savings rate is lowered. This relation between the age structure and human accumulation has not been considered in Horioka (1997) and Kögel (2005), who did not incorporate *endogenous* human capital accumulation in their models. In contrast, we have shown that along the economic development path parents invest more on their children's human capital, thereby reducing the saving rate.[24]

Figure 4a illustrates the predicted time path of the saving rate and the youth dependency ratio of our model economy whose parameters are given at the end of Sect. 3 and the initial values are assumed to be $(k, h) = (0.01, 1)$, while Fig. 4b shows the experience in Japan during the period from 1955 to 2005. The household saving rate is segmented by 63SNA (1955–1979), 93SNA (1980–1995) and 93SNA (1996–2005). Although the time-series data of the household saving rate differ from those in Horioka (1997), the saving rate moves similarly to SR2 in Horioka (1997). DEP is the rate of population at age 19 and under 19 to those of age 20 to 64, and AGE is the rate of population at age 65 and over 65 to those of age 20 to 64 (although DEP/2 and AGE/2 are illustrated in Fig. 4b). REH is the rate of junior-high-school graduates who enter senior high schools, and REC is the ratio of students who enter colleges and universities (in the population at their age 18). As can be seen in Fig. 4b, both the saving rate and the youth dependency ratio have almost the same time trend after most junior-high-school graduates entered senior high schools in the mid-1970s.[25] The experience in Japan seems consistent with the predictions from our model with human capital accumulation.

[24] Savings increase in our model, too. Therefore, it should be noted that our result does not necessarily go counter to the positive effect of the increased savings on TFP growth suggested by Kögel (2005), if we consider R&D innovations dependent on the level of savings, as in Ren and Rangazas (2003). Alternatively, considering R&D innovations dependent on per worker human capital, as in Romer (1990), we may also have a negative relationship between youth dependency and TFP growth in the endogenous growth phase.

[25] In Japan, junior high school education is compulsory, whereas senior high school education is not.

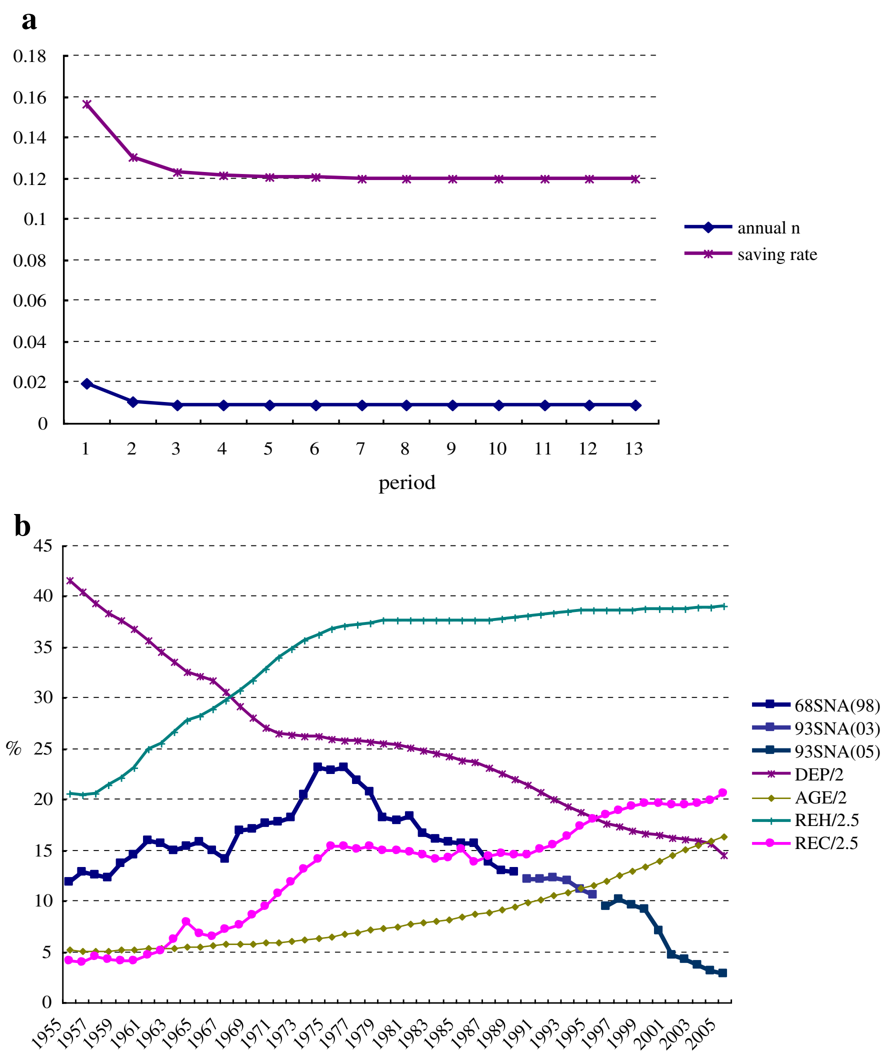


Fig. 4 **a** Youth dependency ratio and saving rate. **b** Experience in Japan (1955–2005). Source: Japanese Cabinet Office, National Disposable Income and its Appropriation Accounts 68SNA (1998), and 93SNA (2003, 2005); and Japanese Ministry of Education, Culture, Sports, Science and Technology, Gakko Kihon Chosa—Annual Statistics (2008)

6 Conclusion

We have examined the development paths of an economy by incorporating the quality-quantity trade-off on the fertility decision of parents and the substitutability between the intergenerational human capital transmission within the family and education paid for by parents into a model populated by identical individuals. There is a threshold wage rate, above which individuals begin to invest in the human capital of their children, while reducing their number. At this point, the economy switches from an exogenous growth path with the engine of physical capital accumulation to an endogenous growth phase with the engine of human capital investment. Although these phases correspond to the regimes which Galor and Weil (2000) referred to as the "Post Malthusian Regime" and "Modern Growth Regime," respectively, we have a regime change caused by changes in fertility decisions due to wage income growth. In the endogenous growth phase toward balanced growth equilibrium, the aggregate saving rate and the youth dependency ratio are both falling (i.e., positively correlated), while the propensity of the working parents to spend on the education of their children will increase. However, whether the economy can change phases or not depends on its structural parameters. If the stationary-state wage rate in the exogenous growth phase does not exceed the threshold, the economy will fall into a development trap, in which individuals do not invest in human capital and the fertility rate remains high.

As to the argument about the relation between youth dependency and the saving rate in the previous section, the TFP growth and the youth dependency ratio may be said to move in opposite directions if we take per worker growth in human capital as TFP growth, as in Shultz (1961). This is similar to the negative relationship between youth dependency and TFP growth as empirically shown in Kögel (2005). However, we did not consider R&D activities explicitly in our model. An extension in this direction would be an interesting issue for future research.

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